ments. In the course of about five minutes from 17h. 8m. to 17h. 13m. there was a swing of 1° 35′ to the east, immediately following a somewhat less rapid swing of 44′ to the west. In the course of about twelve minutes from 1h. 9m. to 1h. 21m. on March 23 there were swings of 76′ to west and 59′ to east, and after a minute's pause the latter swing continued, so that declination at 1h. 28m., when the trace went off the sheet, was 1° 35′ less westerly than it had been nine minutes before. The range actually shown on the sheet, 2° 1′, has seldom been equalled at Kew, and as the trace was off the sheet for twelve minutes it was probably sensibly exceeded.

The vertical force trace was complete, the range of disturbance being about 820γ. The disturbance in that element was fairly normal, the value being enhanced on the afternoon of March 22 from 13h. to 23h., and correspondingly depressed on the morning of March 23 from 0h. 30m. to 7h. The times of maximum and minimum were respectively about 18h. 20m. on March 22 and 4h. 10m. on March 23.

The outstanding features of the disturbance were the size and rapidity of the largest movements, and the separation of two very highly disturbed periods by a comparatively quiet interlude lasting several hours. The disturbance was preceded twenty-seven days before (February 24) by a considerable, but much smaller, disturbance, which was in some respects the antithesis of the later one. It lasted only about ten hours, and the largest movements occurred in the course of the first three hours. C. Chree.

Kew Observatory.

This storm was one of exceptional violence. It commenced suddenly at 9h. 6m. G.M.T. on March 22. The H.F. magnet experienced a sharp positive movement of 37γ, followed immediately by a decrease in force of 41γ ($1\gamma \equiv 10^{-5}$ C.G.S. unit). Similarly, the D. magnet swung sharply to the west and then to the east, the range being 8′ of arc.

After three oscillations on the negative side of the normal, a steady rise of value commenced in the H.F. magnet at 10h. 36m., which lasted until 19h. 10m., when the spot of light began to fall steadily towards the normal value of the force. The general character of the movement was that of one long wave with oscillations superposed upon it, the storm being most violent between 16h. and 19h. 10m. During this period the spot of light passed beyond the limits of registration in a series of rapid oscillations between 16h. 24m. and 16h. 30m., 16h. 46m. and 17h. 12m., 17h. 54m. and 18h. 14m., 18h. 41m. and 18h. 47m., and 18h. 47m. and 18h. 51m. The oscillations became less violent and rapid after 19h. 37m. At 23h. 5m. the spot of light had fallen to a value close to that of the base line, so that the extreme range in this sweep of the curve was from a value greater than 550γ to 52γ.

At 23h. 16m. a rapid oscillatory recovery and increase of value took place, which attained a maximum of 376γ at 23h. 36m. The spot of light then went off the recording drum on the negative edge, crossing the base line on March 23 at 0h. 24m. During the next four hours it was several times beyond the limits of registration on the negative side in a series of rapid oscillations, in which the greatest range exceeded 260γ.

The extreme range in H.F. during the storm was greater than 700γ. This value may be compared with the range on the quiet days during January and February, which had a mean value of 18γ.

At 4h. 20m. the spot of light returned to a positive value, when a series of very rapid shiverings of the needle took place, similar to those which terminated the violent magnetic storm of August 11–12, 1919. These oscillations had a range of about 130γ, and lasted until 8h. 50m. This may be regarded as the end of the violent storm, though the needle continued to be disturbed moderately until midnight of March 25.

The general character of a sinuous S-like curve is well shown on the trace from the vertical force magnet. It crossed the base line at 0h. 44m. on March 23. There was a very rapid oscillation of the needle at 1h. 0m. The spot of light remained below the base line until 6h. 0m., when it gradually rose, with a shivering movement of small amplitude, to its normal value. On the negative side the spot of light was off the recording drum from 1h. 20m. to 2h. 40m. and from 2h. 40m. to 4h. 0m. The extreme range was greater than 900γ, and the greatest positive value was 642γ.

Corresponding to the gradual increase in force in the H.F. and V.F. elements, the declination magnet gradually swung to the west. The maximum disturbance consisted of some rapid swings of the needle between 16h. 24m. and 18h. 48m. The greatest of these was at 17h. 0m., the range being 90′ of arc.

A very remarkable rapid double swing of the needle occurred on March 23 at 1h. 12m. The range of this oscillation was 130′. This corresponds to rapid oscillations in the force elements. The spot of light was now, on the whole, below the base line until 4h. 0m., when there was a rapid movement east and then west between 4h. 15m. and 5h. 0m., with a range of 120′. A series of shivering oscillatory movements then supervened until the end of the storm. The greatest total range in D. during the storm was 160′.

Judging from the three elements, the general movements both in force and in direction were rising with reference to the base line during the daylight hours and falling during the night hours.

The storm was coincident with the appearance of a very great sun-spot group on the sun's disc which appeared between March 16 and 29, and was passing the central meridian on March 22–23. Its mean heliographic latitude was −6°, and it extended from longitude 114° to longitude 150°. It was the biggest group of sun-spots observed since August, 1917, and its disc area, in units 1/5000th of the visible disc, was 34 on March 22.

It was a revival of a similar extended group of spots of large area observed from January 21 to February 3. At the next rotation, February 17–27, this group appeared as an insignificant small spot and dots amidst extensive faculæ. But the magnetic elements began to be disturbed during this second rotation of the spot-group on February 16–17.

Through the kindness of Lt.-Col. Penny, R.A.M.C., the O.C. Queen Mary's Military Hospital, Whalley, in the immediate neighbourhood of Stonyhurst College, I have received the following account of the aurora borealis observed by him in the early morning of March 23:—

"On going out of doors at about 3.15 a.m. I noticed this display, but I do not know how long it had been visible. It was a clear, starlight night at the time. The aurora was exceedingly fine when I first saw it, the best I have ever seen. It consisted of about eight broad beams of light, most of which, except the extreme west and north ones, extended to within 5°–10° of the zenith. The lights extended over about 90°–100° from approximately north-north-east to west by north.

"The beams became pale and brilliant again several times, besides constant slighter variations in intensity. On two or three occasions, within about twenty minutes, most of the beams, more than three-quarters,

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disappeared, leaving one or two longish ones. The colour was mostly white, but sometimes reddish in parts, especially nearly due north.

"A curious feature was an oblique band of light, which came and went across near the summits of the vertical beams. I do not think this was a belt of illuminated cirrus, as its brightness seemed to vary independently of the vertical beams, but it is possible it may have been. The lights had diminished considerably by about 3.45 a.m., but had brightened again, though slightly, when I looked out a few minutes later. I do not know what time the display ended." A. L. CORTIE.

Stonyhurst College Observatory, March 29.

Some Methods of Approximate Integration and of Computing Areas.

THE formulæ which Mr. Percival gives in NATURE for March 18 for approximate integration are well known, but there are one or two points in connection with them which are frequently overlooked, especially by writers of books on mathematics for engineers.

(1) The areas bounded by curves the equations of which are of the form

$$y=a+bx+cx^2+\ .\ .\ .\ +kx^n$$

can be obtained from the values of $2m+1$ equidistant ordinates, not only when $n=2m$, but also when $n=2m+1$. That this is so is seen most easily by taking the origin at the centre of the range of integration and noting that

$$\int_{-h}^{+h}x^{2m+1}dx=0.$$

For example, Simpson's first, or three-ordinate, rule gives the area of the cubic

$$y=a+bx+cx^2+dx^3$$

with perfect accuracy, and for this purpose his second, or four-ordinate, rule is in no way superior.

(2) By a very small change in one of the coefficients Weddle threw the seven-ordinate formula (No. 6 in Mr. Percival's letter) into the very convenient form

$$\mathrm{A}=\frac{3h}{10}\Big[y_1+y_3+y_5+y_7+5(y_2+y_6)+6y_4\Big].$$

The loss of accuracy which the change involves is exceedingly small.

(3) Formulæ based upon the assumption that the boundary curve can be represented by an equation of the form above stated give unsatisfactory results when the actual boundary has tangents at right angles to the x-axis. This is really the reason why none of the results obtained by Mr. Percival in applying his formulæ to the quadrature of a circle possess a higher degree of accuracy than that represented by the admission of errors of the order of 1 per cent.

If we suppose the curve to cut the axis of x at right angles at the origin, it is better to assume that it can be represented by $y=ax^{\frac{1}{2}}+bx$ in the neighbourhood of that point.

If y_1, y_2 be the ordinates at $x=h$, $x=2h$, the area bounded by the curve, the axis of x and the ordinate y_2 is given by

$$\mathrm{A}=\frac{4h}{15}\Big[4\sqrt{2}y_1+y_2\Big]$$

The much higher degree of accuracy resulting from the employment of this formula may be illustrated by applying it to Mr. Percival's example of the quadrant of a circle.

The seven ordinates are:—

$y_0=0$
$y_1=0{\cdot}5527708$
$y_2=0{\cdot}7453560$
$y_3=0{\cdot}8660254$
$y_4=0{\cdot}9428090$
$y_5=0{\cdot}9860133$
$y_6=1$

Using the above formula to find the area between the ordinates y_0 and y_2, and Simpson's first formula for the part between y_2 and y_6, we obtain the value 0·7853871. The true value is 0·7853982; hence the percentage error is only 0·0014, which compares very favourably with the errors ranging from 0·8 to 1·34 per cent. obtained by using the usual formulæ for the whole range.

Mr. Percival's example clearly shows that when the curve has a tangent at right angles to the axis, no material reduction in error is attained by using formulæ with a larger number of ordinates. The use of Simpson's formula over ordinary ranges and of the formula given above in the neighbourhood of such tangents will prove much less laborious and far more accurate. J. B. DALE.

King's College, Strand, March 22.

IN NATURE of March 18 Mr. A. S. Percival gives an example (the quadrant of a circle) in which Simpson's rule (sometimes called his first rule) is more accurate than the "three-eighths" rule, and he remarks: "This result is curious, and shows that a small arc of a circle approaches more nearly to a small arc of a parabola than to a small arc of any cubic curve." Permit me to point out that this inference is not valid, and is based on the almost universal illusion that Simpson's rule is correct to the second order only, *i.e.* for the parabola

$$y=a+bx+cx^2.$$

It is easy to show by simple integration that Simpson's rule holds to the *third* order, *i.e.* for *all* cubics of the form

$$y=a+bx+cx^2+dx^3,$$

passing through the three chosen points. It is thus precisely accurate, not only for the parabola, but also for a singly *infinite* number of curves passing through the three points, even if an inflexion occurs.

One would therefore expect (which I believe to be the case) that where both rules can be applied (*e.g.* if there are seven ordinates) Simpson's rule would be more accurate than the "three-eighths" rule, which is precisely true only for a single curve passing through four consecutive points.

In some cases, when the gradient is not rapid, Simpson's rule is highly accurate. Dr. Lamb ("Infinitesimal Calculus," p. 278) gives an example in the evalution of π to six decimal places from the equation

$$\int_0^1\frac{dx}{1+x^2}=\frac{\pi}{4}$$

by taking ten equidistant values for x, but he does not notice the illusion to which I refer. I am surprised that such a simple and easily tested truth should so long have escaped the notice of many expert mathematicians. R. A. P. ROGERS.

Trinity College, Dublin, March 20.

Gravitational Deflection of High-speed Particles.

THE result mentioned by Mr. Leigh Page and verified by Prof. Eddington (NATURE, March 11, p. 37), that the gravitational effect on a particle travelling radially is a repulsion if the speed exceeds $1/\sqrt{3}$ times the light-velocity, is given by Hilbert in the *Göttinger Nachrichten* for 1917. The same paper contains interesting remarks on the path of a particle or light-pulse moving spirally round the gravitation centre.

Hymers College, Hull. H. G. FORDER.

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